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PITTSBURGH UNIV PA DEPT OF ELECTRICAL ENGINEERING
APPROXIMATION METHODS IN MULTIDIMENSIONAL FILTER DESIGN.(U)
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(Continued)

20. Abstract continued

greater than twice the sum of multiplicative complexities of two 1-D DHT's. Subsequently, the 1-D matrix Pade approximation problem via a three-term recursive computation scheme was tackled as a prelude to the solution of 2-D and n-D cases. Specifically, given a 1-D matrix power series, it was shown that a recurrence relation relates the $[L+1/M+1]$, $[L/M]$, $[L-1/M-1]$ order Pade approximants, which are guaranteed to exist provided a certain rank condition is satisfied by characterizing matrices possessing block-Hankel structure. Attention to stability, algebraic computational complexity and approximation were necessary because efficient implementation of stable recursion is desired.

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b. STATEMENT OF WORK

The key objective of the research has been to make fundamental contributions towards the design of multidimensional recursive digital filters and linear processors. The major emphasis was planned for:

- (a) development of new multidimensional stability test procedures as well as stability tests
- (b) the two-dimensional discrete Hilbert transform (DHT) and algebraic computational complexity aspects for its efficient implementation in problems of 2-D stabilization as well as 2-D filtering, and
- (c) the approximation of a prescribed sequences of impulse responses that could characterize either a 2-D single-input single-output filter or a 1-D multi-input multi-output (MIMO) filter as a prelude to the approximation of multivariable multidimensional impulse response array sequences.

It might be noticed that the above three stated objectives are closely linked and could demand attention at various stages of a successful design. Most of the stability procedures developed to-date are applicable to the bounded-input bounded-output (BIBO) type and tend to become computationally tedious, especially with increase in the number of dimensions of the filter. The need for new procedures, therefore, is not over. Also, the BIBO stability criterion may not be sufficient in many applications. For example, an n -dimensional linear processor (noncausal, in general) is stable if its response decreases exponentially in all $2-n$ directions. This can be translated into the absence of zeros of the denominator polynomial on the polydisc distinguished boundary. Thus, the potentially trouble-causing non-essential singularities of the second kind on the polydisc distinguished boundary in BIBO stability are automatically eliminated. Applications of this type of stability condition include 2-D image processing, 3-D image reconstruction from 2-D projections and real time image processing of moving objects. Therefore, a goal of the research has been to develop tests for determination of the presence or absence of multivariate polynomial

zeros on the unit polydisc distinguished boundary.

After application of the stability tests, if a filter is found unstable, one might like to stabilize the unstable filter without any appreciable distortion in the filter's desired amplitude response spectrum. The infeasibility, in general of stabilizing a 2-D filter using extensions of known 1-D procedures so that the amplitude response exactly matches that of an unstable filter has been demonstrated. In the absence of, as yet, a general 2-D stabilization procedure, it appears that stabilization via 2-D DHT, whenever applicable, cannot be overlooked. This and other potential applications including 2-D lowpass digital filtering provided the motivation for planning research on the 2-D DHT. The goal has been to study the special properties, if any, of the 2-D DHT impulse response operator and exploit to advantage the algebraic structure of the system in order to be able to implement efficiently the 2-D discrete Hilbert transformer. Recent results on the minimal multiplicative complexity have been planned for use in the present application.

The approximation of functions of several variables is probably the branch of the subject offering the toughest challenge. Research into stability and stabilization problems were planned because rational approximants must satisfy stability constraints prior to recursive implementation. Though much of the one variable theory can be extended without very little modification, fundamental difficulties often occur in the multidimensional arena. For example, there are no Chebyshev sets in many dimensions so that minimax approximation is not necessarily unique. Non-uniqueness does not necessarily disturb one in practical applications. The problem of uniqueness, however, is closely connected with that of characterization, and the latter is of much greater practical importance. Therefore, multidimensional

minimax approximation has mostly been avoided in practice. After careful study of the only available results in the area along with the extension of 1-D differential correction scheme and some hybrid approximation schemes, it was decided that in the context of multidimensional digital filter design, development of recursive schemes for implementing the Pade approximation theory might be the most appropriate. Therefore, research into the theory of existence, non-uniqueness and recursive computation of 2-D scalar and 1-D matrix Pade approximants was planned as a prelude to the development of the corresponding results required in the design of multivariable multidimensional digital filters with stability constraints.

c. STATUS OF RESEARCH EFFORT

The overall objective of the research has been to contribute towards the approximation problem in the design of stable multidimensional recursive filters and linear processors. The key factors considered in this research towards the fulfillment of the final objectives are development of new multidimensional stability tests, the approximation of a power series in several variables by rational approximants and the algebraic computational complexity problems that must be attended to for efficient implementation.

1. Multidimensional Stability Tests: Over the last few years multidimensional systems research has progressed along different directions as described in [1], [2]. The topic of stability tests for multidimensional filters has been the subject of intensive research over the past several years and an up-to-date exposition of research on the subject is available in [3]. Most of the tests have centered around bounded-input bounded-output type of stability, while an n -dimensional linear processor is stable if its impulse response decreases exponentially in all $2-n$ directions. It can be shown that every rational processor transfer function has a unique stable expansion if the denominator polynomial $B(z_1, z_2, \dots, z_n)$ in the complex variables z_1, z_2, \dots, z_n satisfies the condition given in (1).

$$B(z_1, z_2, \dots, z_n) \neq 0, \text{ on } |z_1| = |z_2| = \dots = |z_n| = 1 \quad (1)$$

Applications include 2-D image processing, 3-D image reconstruction from 2-D projections and cross-sections, real time image processing of moving objects, repetitive infinite networks and possibly in tests for existence of Green's function with vanishing infinite boundary conditions in linear partial difference equations. The present research contributed to the solution for the test of condition in (1), which did not exist except for $n = 1$ case.

Two indirect and one direct procedure to test a prescribed multivariate polynomial for zeros on a polydisc distinguished boundary substantiate constructively the existence results from elementary decision algebra [4]. The exact methods implementable, if necessary, with arbitrary precision, complement the numerical method involving DFT computations. In many cases, the latter method can give satisfactory result with speed while in some cases involving high coefficient sensitivities (not uncommon in multi-dimensional problems) the procedures given in the paper [5], resulting from this research, are more suited. In this paper, the first indirect test formulation relates the present test to an already developed global positivity test on a polynomial in several real variables [6] while the second relates the present test to an already developed local positivity test package [7], [8]. A direct test is then formulated for the problem under consideration. Detailed discussions are given for the $n = 2$ case with adequate attention to the various singular cases that might occur in the implementation of the procedure. The feasibility of implementing the test via rational operations is shown for the $n = 2$ case. Parallels for the $n > 2$ case are briefly given, and the details with illustrative example for the direct test along with its limitations when $n > 2$, is available in [9]. The important research results obtained in the area have been reported in:

N. K. Bose and S. Basu, "Test for polynomial zeros on polydisc distinguished boundary," IEEE Transactions on Circuits and Systems Special Issue on "Mathematical Foundations of System Theory," Vol., 25, Sept., 1978, pp. 684-693.

Another n -dimensional filter bounded-input bounded-output stability test was developed during the course of the research as a follow-up of a new stability test and its implementation for two-dimensional filters, advanced by the principal investigator earlier [10]. It was shown how after a single

dimensional stability test, the final result obtained for the n-D case requires the testing of a single variable, two variable, ... (n-1) variable polynomials (in real variables) for local positivity in $-1 \leq x_1 \leq 1$, $-1 \leq x_i \leq 1$, $i = 1, 2, \dots$, $-1 \leq x_{i-1} \leq 1$, $i = 1, 2, \dots$, $n-1$. These polynomials can be conveniently generated in the sense that all remaining polynomials can be obtained by specializing to zero relevant variables at individual stages in the recursive scheme used in the computation of the (n-1) variable polynomial. The research results obtained in this connection were published in:

N. K. Bose, "Implementation of a new stability test for n-D filters," IEEE Transactions on Acoustics, Speech and Signal Processing, Vol., 27, February, 1979, pp. 1-4.

2. 2-D DHT and Computational Complexity Aspects: It was shown that the 2-D discrete Hilbert transform (DHT) operator may be expressible as:

$$H(i_1, i_2) = \frac{1}{N_1 N_2} \{ [1 - (-1)^{i_1}] I_1 + [1 - (-1)^{i_2}] I_2 - [1 - (-1)^{i_1+i_2}] I_1 I_2 \}$$

where

$$j = \sqrt{-1}, \quad w_1 = e^{\frac{j2\pi}{N_1}}, \quad w_2 = e^{\frac{j2\pi}{N_2}} \quad \text{in}$$

$$I_1 = \frac{w_1^{i_1} - (-1)^{i_1}}{w_1^{i_1} - 1}, \quad I_2 = \frac{w_2^{i_2} - (-1)^{i_2}}{w_2^{i_2} - 1}$$

In the standard vector form representation,

$$\text{vec } \{PI\} = \underline{H} \text{vec } \{PR\} \quad (\text{or } \underline{Y} = \underline{H} \underline{X})$$

of the 2-D convolution,

$$PI(i_1, i_2) = \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} PR(k_1, k_2) H(i_1 - k_1, i_2 - k_2)$$

for $i_1 = 0, 1, 2, \dots, N_1 - 1$, $i_2 = 0, 1, 2, \dots, N_2 - 1$,

the $N_1 N_2 \times N_1 N_2$ matrix \underline{H} has a block-circulant structure. Each of the $N_2 \times N_2$ element matrices in \underline{H} possesses a circulant-like structure. This algebraic structure was exploited to arrive at the main results summarized in lemma 1 and theorem 1 below.

Lemma 1. The 2-D DHT of an $N_1 \times N_2$ input array can be computed using only,

$$\alpha(N_1, N_2) = 2\alpha(N_1) + 2\alpha(N_2)$$

multiplications, where $\alpha(N)$ is the number of multiplications required in the computation of the 1-D DHT of a N -point input vector.

Theorem 1. The 2-D DHT of an $N_1 \times N_2$ input array can be computed using only

$$\alpha(N_1, N_2) = 4(N_1 + N_2) - 2(\phi(N_1) + \phi(N_2))$$

multiplications (provided multiplication by any fixed element of the field is not counted as a multiplication), where $\phi(N)$ is the number of distinct irreducible factors over G of the polynomial in u , namely $u^N - 1$.

In the absence of, as yet, a general 2-D stabilization procedure, it appears that stabilization via 2-D DHT, whenever applicable, cannot be overlooked. This and other potential applications including 2-D lowpass digital filtering provided motivation towards the obtaining of results on the two-dimensional DHT and computational complexity aspects in its implementation as an integral part of the topic of the present research. The details of the research just outlined will appear in:

N. K. Bose and K. A. Prabhu, "2-D DHT and computational complexity aspects in its implementation," accepted for publication in IEEE Transactions Acoustics, Speech and Signal Processing, August, 1979.

In view of the important role of finite field theory in the efficient implementation of developed algorithms, attention was directed towards some other aspects, which show strong potentialities for future engineering

applications and offer scope for further investigations and developments. In texts, a procedure to obtain the number of distinct irreducible monic polynomials of prescribed degree m in a single variable x_1 , having coefficients over a finite field F of q elements, has been given. This has been done through the use of the concepts of generating functions and Moebius inversion. The result has been of use in multiple error-correcting code construction as well as in other problems of algebraic coding theory. In the present research a procedure to determine the number of distinct irreducible polynomials of prescribed degrees, m_1, m_2, \dots, m_n in the independent variables x_1, x_2, \dots, x_n , respectively, having coefficients over a finite field of q elements was obtained. It was seen that certain interesting distinction from known results occur in the resolution of this problem—a fact often encountered in the transition from the single dimensional to the multidimensional case. Though the results arrived at have their validity for an arbitrary n , in the research conducted, for the sake of brevity in exposition attention has been especially concentrated on the $n = 2$ or the 2-D case as reported in the preprint:

K. A. Prabhu and N. K. Bose, "Number of irreducible q -ary polynomials in several variables with prescribed degrees," accepted for publication in IEEE Transactions of Circuits and Systems.

3. Approximation theory results: The 1-D scalar Pade approximation theory has been widely used in problem of theoretical physics [11], [12], numerical analysis [13], and electrical engineering especially because of its close link to the Prony method [14] and the theory of continued fractions [15]. The necessity for generalizing the 1-D Pade scalar theory to the 1-D($m \times n$) matrix case was discussed in the setting of the minimal partial realization problem [16]. The feasibility for recursive computation of the 1-D matrix

Pade approximants in the context of minimal partial realization theory was shown in [17]. The matrix Pade approximation theory in the setting of a noncommutative algebra with a view towards application in scattering physics, is also available in [1, ch. 21], [12, pp. 19-44]. More applications of the 1-D matrix Pade theory can occur in multiport network synthesis and in the design of multi-input multi-output digital filters. The present research provided an efficient recursive computational scheme for the 1-D matrix Pade approximants as a prelude to 2-D and n-D results required in, among other problems, in the design of multivariable multidimensional digital filters. The main contribution of the present research was the demonstration of the existence of a recurrence relation, using which the $[^{L+1}/_{M+1}]$ order 1-D matrix Pade approximant can be computed from the $[^L/_M]$ and $[^{L-1}/_{M-1}]$ order 1-D matrix Pade approximants to a prescribed matrix power series $T(z)$ of order $m \times n$. It is also possible to obtain the matrix polynomial $B_{M+1}(z)$ associated with the $[^{L+1}/_{M+1}]$ order approximant by solving a system of linear equations, characterized by a block Hankel matrix of order $\{(M+1)m\} \times \{(M+1)n\}$. In [18], for the $m = n = 1$ case a solution scheme requiring $O(M^2)$ operations has been advanced and the feasibility for extension to the more general block matrix case has only been cited. Therefore, computational complexity comparison between the two schemes can be carried out as a separate project. The advantage of the technique presented here appears to lie in the use of the recurrence relation established, where the order of matrices, whose inverses or right inverses have to be computed, is independent of the order of approximants. The details of these results are available in:

N. K. Bose and S. Basu, "Theory and recursive computation of 1-D matrix Pade approximants," Proceedings of 1978 Conference on Dec. and Control, pp. 653-657. Also an article has been completed for submission to a journal.

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d. PUBLICATIONS IN TECHNICAL JOURNALS

1. N. K. Bose and S. Basu, "Tests for polynomial zeros on a polydisc distinguished boundary," IEEE Transactions on Circuits and Systems Special Issue on "Mathematical Foundations of System Theory," vol., 25, Sept., 1978, pp. 684-693.
2. N. K. Bose, "Implementation of a new stability test for n-D filters," IEEE Transactions on Acoustics, Speech and Signal Processing, vol., 27, Feb., 1979, pp. 1-4.
3. N. K. Bose and K. A. Prabhu, "2-D discrete Hilbert transform and computational complexity aspects in its implementation," IEEE Transactions of Acoustics, Speech and Signal Processing, accepted for publication in August, 1979.
4. K. A. Prabhu and N. K. Bose, "Number of irreducible q-ary polynomials in several variables with prescribed degrees," IEEE Transactions of Circuits and Systems, accepted for publication in 1979.
5. N. K. Bose and S. Basu, "Theory and recursive computation of 1-D matrix Pade approximants," Proc. of 1978 IEEE Conference on Decision and Control, pp. 653-657. Also has been prepared for journal publication.

BOOK

6. N. K. Bose, ed., "Multidimensional Systems: Theory and Applications," IEEE Press, N.Y., 1979.

e. RESEARCH PERSONNEL

1. Dr. N. K. Bose, Professor of Electrical Engineering and Mathematics served as the Principal Investigator throughout the period of report extending from January 1, 1978, to January 31, 1979. Worked and guided work on problems of multidimensional stability, stabilization, approximation and algebraic computational complexity in implementation.
2. Mr. S. Basu, a Graduate Student Researcher, worked on research supported by the grant for the complete period. He completed a M.S. thesis entitled, "Theory and applications of a direct test procedure for polynomial zeros on polydisc distinguished boundary," at the University of Pittsburgh in April, 1978. Subsequent to that, he worked on the approximation problem in the design of multidimensional digital filters.
3. Mr. K. A. Prabhu, worked for five months as a Research Fellow and one month as a Graduate Student Researcher. His efforts were concentrated primarily towards the algebraical computational complexity aspects in the implementation of stability and approximation algorithms occurring in the design of multidimensional filters. Approximately two man-months of support were also provided by Graduate Student Researchers, Mr. Wen and Mr. John W. Loney. Mr. Wen began to look into the efficient implementation of 2-D algorithms for signal processing while Mr. John W. Loney participated for a month on discussions involving scopes for multidimensional minimax approximation which was, however, discarded in favor of Pade approximants with stability constraints as more suited for the type of research undertaken here.

f. INTERACTIONS OF PRINCIPAL INVESTIGATOR, DR. N. K. BOSE

1. Invited to be the Organizer and Session Chairman of a Special Session on "Multidimensional Systems," at the 1978 International Symposium on Circuits and Systems held at New York City, May 16-19, 1978. Also presented a paper entitled:

"Test for polynomial zeros on polydisc distinguished boundary and consequences,"

at the Symposium.
2. Invited for collaborative research on multidimensional wave digital filters for one month at Ruhr Universität, West Germany in 1978. Visit was sponsored by Deutscher Akademischer Austauschdienst. Took part in numerous technical discussions with Professor Fettweis and his assistants - primarily Herr Klaus Meerkotter and also, to some extent, Herr Lennarz and Herr Linnenberg. The topics of discussion centered primarily around various problems associated with the synthesis of two-dimensional wave digital filters, as well as the multidimensional stability tests recently developed by the author, and the importance of approximation in the context of 2-D filter design.
3. Was an organizer and participant in the National Science Foundation sponsored Regional Conference on Algebraic Complexity of Computation and its Applications to Problems in Engineering and Computer Sciences, held at the University of Pittsburgh during Aug. 21-25, 1978. Introduced here the properties of 2-D Hilbert transformer and subsequently employed aspects of computational complexity theory, presented by S. Winograd, to efficiently implement the 2-D discrete Hilbert transform in 2-D stabilization and 2-D digital filtering problems.

4. Invited to present a paper in the Special Session on "Multidimensional Systems" organized by Dr. M. Morf of Stanford University and Dr. B. Levy of the Massachusetts Institute of Technology at the Conference on Decision and Control held at San Diego, Jan., 10-12, 1979. The invited talk presented at the Conference was entitled, "Multidimensional Systems Theory: Present state as an indicator of future prospects," and a portion of the talk was published in the Conference Proceedings as, "Multidimensional Systems Theory: 1-D Matrix Pade Approximants."
5. The paper "Sum and product separabilities of multivariable functions and applications," by S. Chakrabarti, N. K. Bose and S. K. Mitra was included in the book entitled, Two-Dimensional Digital Signal Processing, published by Academic Press in 1978 as volume 20 in Benchmark Papers in Electrical Engineering and Computer Science.

g. NEW RESULTS AND SPECIFIC APPLICATIONS RESULTING
FROM RESEARCH EFFORT

1. The two indirect and one direct test procedures for zeros on the distinguished boundary of the polydisc provide tests for stability of a n -dimensional linear processor which is required to have its impulse response decrease exponentially in all $2-n$ directions. Applications include 2-D image processing, 3-D image reconstruction from 2-D projections, real time image processing of objects or targets in motion, repetitive infinite networks and tests for existence of Green's function with vanishing infinite boundary conditions in linear partial difference equations. The indirect tests dwell on the multivariate polynomial global positivity and local positivity test procedures previously developed by the principal investigator.
2. The new BIBO stability test procedure for multidimensional recursive filters has a natural potential for application. The efficiency in implementation and, consequently, the popularity of the procedure will increase with the improvement of multivariate polynomial local positivity test methods.
3. The demonstration of the fact that the multiplicative complexity for implementation of a 2-D DHT is not greater than twice the multiplicative complexities of only two 1-D DHT's will be a strong motivation behind the use of 2-D DHT in stabilization and low-pass digital filtering problems.
4. The development of a recursive enumeration procedure for the number of irreducible q -ary polynomials in several variables with prescribed degrees is a fundamental contribution in multidimensional signal

processing over finite fields. Scopes for application are included in signal processing, system identification, pseudo-random noise generation, q -ary array synthesis and multidimensional codeword generation.

5. Finally, the development of a three-term recursion procedure for computation of 1-D matrix Pade approximants is likely to be of use in several areas besides MIMO digital filtering. This fundamental contribution can be directly applied to the minimal partial realization problem, identification of linear dynamical systems from time-domain measurements, the scattering problem in high energy theoretical physics, and possibly in the synthesis of multiport distributed networks by lumped approximants.